

Hunter College High School
Mathematics Department
Re: A.P. Calculus summer work

June 18, 2010

Dear Budding Calculus Student:

Welcome to Advanced Placement Calculus! To help you to know whether or not you are prepared for this rigorous—but fun—course, the attached packet of exercises represents the bare minimum of mathematical prowess that you should have. **This packet is required**; you must turn it in within the first two class periods next year, with a test which your teacher may administer in the first two weeks. The purpose of this summer work is to emphasize the necessity of this prerequisite knowledge for your success in Calculus. You will utilize these ideas as a launch pad for the material that will be covered. In addition to a need for solid algebra skills, your next year of math will rely heavily on functions and their notation, trigonometry, and logarithms. This packet should not be overly challenging, but if you have trouble, you should seek information on the Internet or from your previous class notes.

Given that this is an A.P. course, the pace will be fast. This makes homework completion that much more essential. If you fall behind on the homework, you will likely struggle in the course. It is also important to note that the purpose of the class is to prepare you for the Advanced Placement Test in May. It is expected that all students who take the course will also take the A.P. Test. Even after this examination, there are a few topics that the College Board assumes will be covered. These topics will round out your knowledge of Calculus and ensure that you are prepared for your experience in college mathematics the following year.

If you lose this packet, it will be posted on the math department's section of the Hunter College High School website at <http://www.hchs.hunter.cuny.edu/index.php/departments/mathematics/>.

Thank you and good luck! Enjoy the summer.

Show all work on a separate sheet of paper neatly and clearly, and write all answers on the answer sheet provided.

Simplify.

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12} \quad 2. \frac{x^3 - 8}{x - 2} \quad 3. \frac{5 - x}{x^2 - 25} \quad 4. \frac{9 - x^{-2}}{3 + x^{-1}} \quad 5. \frac{1}{x+h} - \frac{1}{x} \quad 6. \frac{\frac{2}{x^2}}{\frac{10}{x^5}} \quad 7. \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$$

$$8. \frac{2x}{x^2 - 6x + 9} - \frac{1}{x+1} - \frac{8}{x^2 - 2x - 3} \quad 9. x^{\frac{3}{2}} \left(x + x^{\frac{5}{2}} - x^2 \right) \quad 10. \frac{4xy^{-2}}{12x^{\frac{1}{3}}y^{-5}}$$

$$11. e^{\ln 3} \quad 12. e^{(1+\ln x)} \quad 13. \ln 1 \quad 14. \ln e^7 \quad 15. \log_3 \left(\frac{1}{3} \right) \quad 16. \log_{1/2} 8$$

$$17. \ln \frac{1}{2} \quad 18. e^{3 \ln x} \quad 19. 27^{2/3} \quad 20. (4a^{5/3})^{3/2} \quad 21. (5a^{2/3})(4a^{3/2}) \quad 22. \frac{3(n+1)!}{5n!}$$

$$23. \log 8 - \log 2 \quad 24. \log_2 5 + \log_2 \frac{1}{5} \quad 26. \ln e^3 + \ln 1$$

Factor each expression.

$$27. x^3 + 27 \quad 28. 4x^3 - 19x^2 - 5x \quad 29. (2x - 3)^3(x + 1) + (x - 3)(2x - 3)^2$$

$$30. (3x - 2)^{-4}(x + 3) + (x + 3)^2(3x - 2)^{-3}$$

Complete the following identities.

$$31. \sin^2 x + \cos^2 x = \quad 32. 1 + \tan^2 x = \quad 33. \cot^2 x + 1 =$$

$$34. \cos 2x = \quad 35. \sin 2x =$$

Solve for z:

$$36. 4x + 10yz = 0 \quad 37. y^2 + 3yz - 8z - 4x = 0$$

Expand and simplify

$$38. \sum_{n=0}^4 \frac{n^2}{2} \quad 39. \sum_{n=1}^3 \frac{1}{n^3} \quad 40. (x + y)^3$$

Write the equation of the line.

41. with slope -2, containing the point (3, 4)

42. containing the points (1, -3) and (-5, 2)

43. with slope 0, containing the point (4, 2)

44. perpendicular to the line in problem #41, containing the point (3, 4)

Without a calculator, determine the exact value of each expression.

45. $\sin 0$ 46. $\sin \frac{\pi}{2}$ 47. $\sin \frac{3\pi}{4}$ 48. $\cos \pi$ 49. $\cos \frac{3\pi}{4}$ 50. $\cos \frac{\pi}{3}$ 51. $\tan \frac{7\pi}{3}$
52. $\tan \frac{\pi}{6}$ 53. $\tan \frac{2\pi}{3}$ 54. $\cos(\sin^{-1} \frac{1}{2})$ 55. $\sin^{-1} \sin(\frac{7\pi}{6})$

For each function, determine its domain and range.

56. $y = \sqrt{x-4}$ 57. $y = \sqrt{x^2-4}$ 58. $y = \sqrt{4-x^2}$ 59. $y = \sqrt{x^2+4}$

Write as a single equation in terms of x and y.

60. $\begin{cases} x = t + 1 \\ y = t^2 - t \end{cases}$ 61. $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

62. Find the remainder on division of: $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

63. Evaluate $\frac{f(x+h) - f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

Determine all points of intersection.

64. $y = x^2 + 3x - 4$ and $y = 5x + 11$ 65. $y = \cos x$ and $y = \sin x$ in the first quadrant.

Solve for x, where x is a real number.

66. $x^2 + 3x - 4 = 14$ 67. $\frac{x^4 - 1}{x^3} = 0$ 68. $(x-5)^2 = 9$ 69. $2x^2 + 5x = 8$

70. $(x+3)(x-3) > 0$ 71. $x^2 - 2x - 15 \leq 0$ 72. $12x^2 = 3x$ 73. $27^{2x} = 9^{x-3}$

74. $\sin 2x = \sin x, 0 \leq x \leq 2\pi$ 75. $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$

76. $\log x + \log(x-3) = 1$ 77. $\log_{81} x = -\frac{1}{3}$ 78. $\ln e^x = 4$

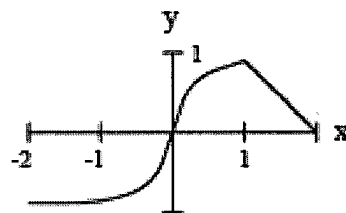
Find the following if $f(x) = x^3 + 1$, $g(x) = x^2 - 2$, and $h(x) = x + 3$.

79. $f(g(2))$ 80. $g(h(x))$ 81. $f(x+h)$ 82. $\frac{g(t+h) - g(t)}{h}$

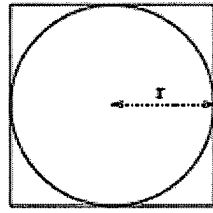
The graph of the function $y=f(x)$ is given as follows:

Determine the graphs of the functions:

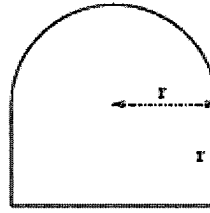
83. $f(x+1)$ 84. $f(-x)$ 85. $|f(x)|$ 86. $f(|x|)$



87. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a)



(b)

88. Find a formula for the perimeter of a window of the shape in the picture (b) above.

89. A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

90. Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50km/hour. How far apart are they two hours later?

91. A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

Graph each function. Give its domain and range.

92. $y = \sin x$

93. $y = e^x$

94. $y = \sqrt{x}$

95. $y = \sqrt[3]{x}$

96. $y = \ln x$

97. $y = |x + 3| - 2$

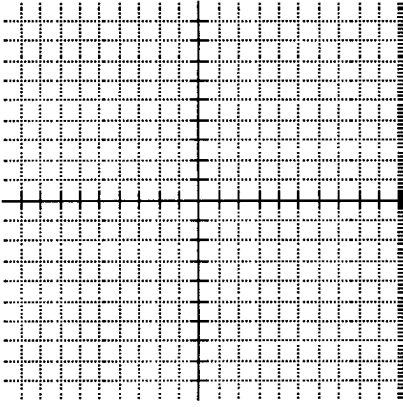
98. $y = \frac{1}{x}$

99. $y = \begin{cases} x^2, & x < 0 \\ x + 2, & 0 \leq x \leq 3 \\ 4, & x > 3 \end{cases}$

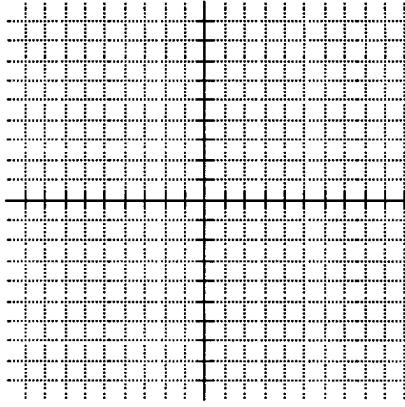
1.	16.	31.
2.	17.	32.
3.	18.	33.
4.	19.	34.
5.	20.	35.
6.	21.	36.
7.	22.	37.
8.	23.	38.
9.	24.	39.
10.	25.	40.
11.	26.	41.
12.	27.	42.
13.	28.	43.
14.	29.	44.
15.	30.	45.

46.	61.	76.
47.	62.	77.
48.	63.	78.
49.	64.	79.
50.	65.	80.
51.	66.	81.
52.	67.	82.
53.	68.	86.
54.	69.	87.
55.	70.	88.
56.	71.	89.
57.	72.	90.
58.	73.	91.
59.	74.	
60.	75.	

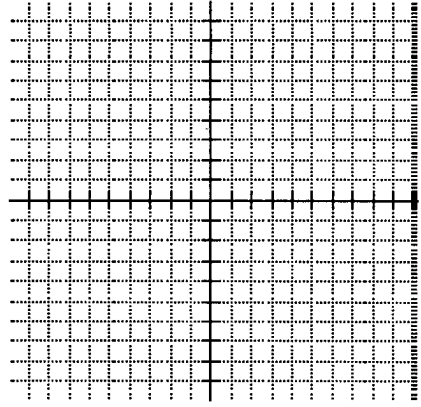
83.



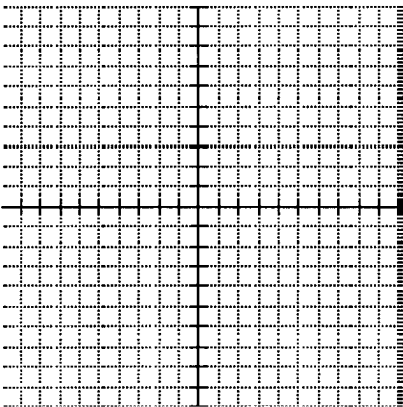
84.



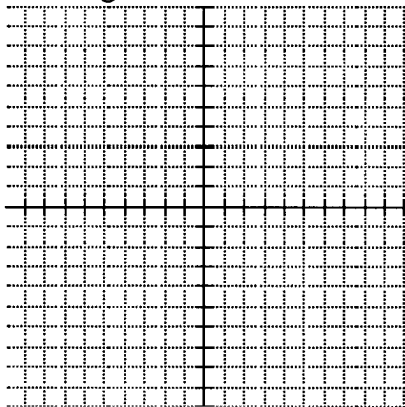
85.



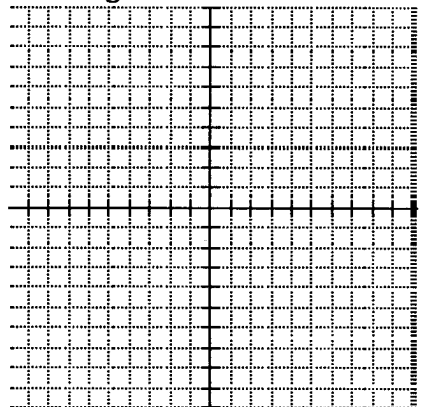
86.



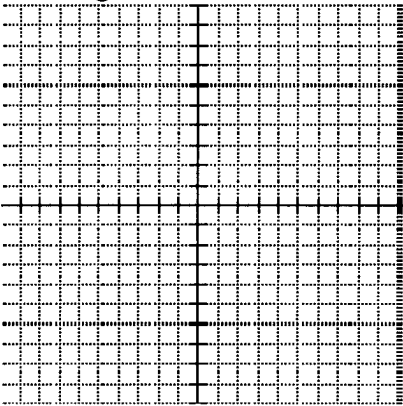
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Range:



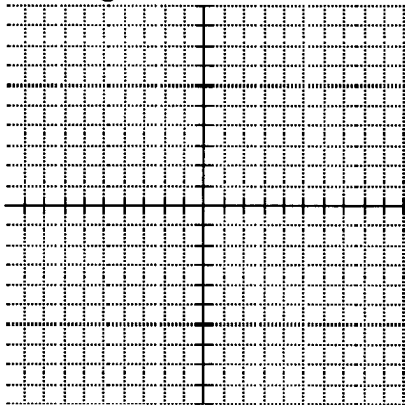
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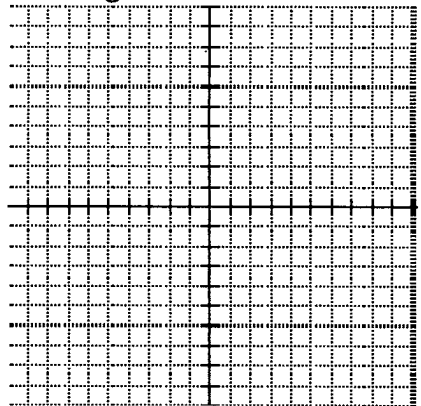
94. Domain:
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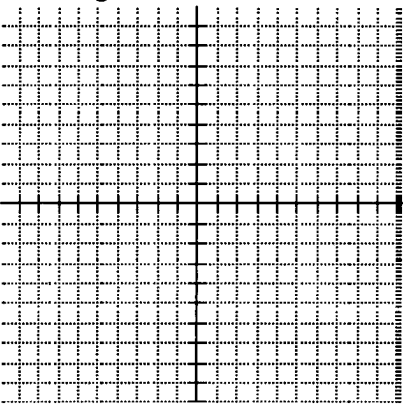
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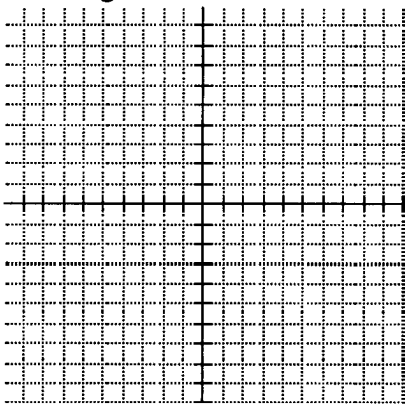
96. Domain:
Range:



97. Domain:
Range:



98. Domain:
Range:



99. Domain:
Range:

